

Direct Stress & Strain.

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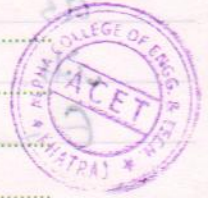
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TOPIC:



- ① A mild steel bar 1m long & 20 mm dia is subjected to an axial tensile force 62.83 kN. If modulus of elasticity $E = 2 \times 10^5 \text{ N/mm}^2$, find: (i) stress (ii) strain (iii) Elongation (iv) final length.

→ Data Given :-

$$P = 62.83 \text{ kN} = 62.83 \times 10^3 \text{ N}$$

$$L = 1 \text{ m} = 1000 \text{ mm}$$

$$d = 20 \text{ mm}, \therefore A = \frac{\pi}{4} \times 20^2 = 314.15 \text{ mm}^2$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\rightarrow \sigma = P/A = \frac{62.83 \times 10^3}{314.15} = 200 \text{ N/mm}^2$$

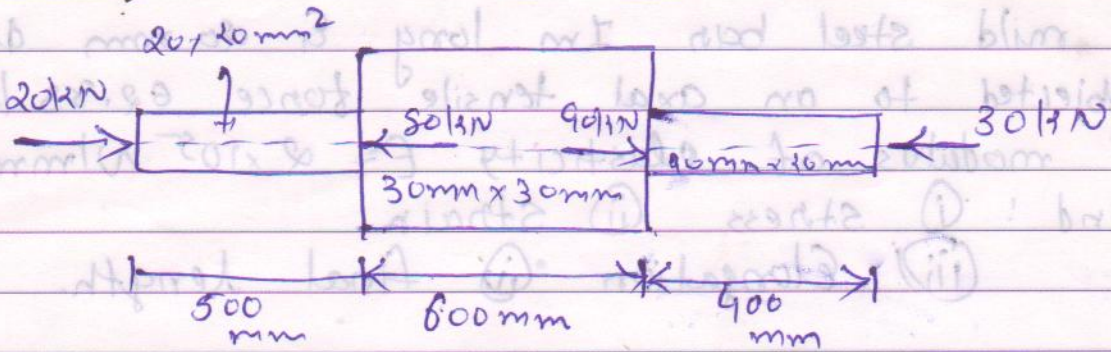
$$\rightarrow \text{Strain} : \epsilon = \sigma/E = \frac{200}{2 \times 10^5} = 0.001$$

$$\rightarrow \text{Elongation} : \Delta L = \frac{PL}{AE} = \frac{(62.83) \times 10^3 \times 1000}{314.15 \times 2 \times 10^5}$$

$$= 1 \text{ mm} \therefore \text{increase}$$

$$\rightarrow \text{Final length} = 1000 + 1 = 1001 \text{ mm}$$

② A bar of Al is subjected to axial forces as shown in fig. Find change in length, its nature & final length of bar.
 $E = 1.5 \times 10^5 \text{ N/mm}^2$.

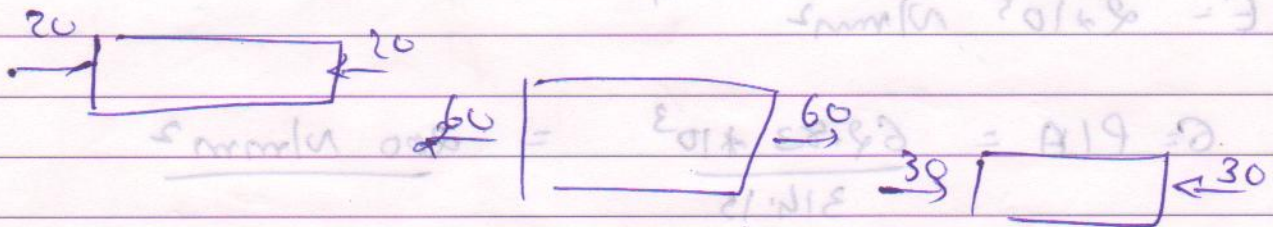


$$A_1 = 20 \times 20 = 400 \text{ mm}^2, \quad l_1 = 500 \text{ mm}$$

$$A_2 = 30 \times 30 = 900 \text{ mm}^2, \quad l_2 = 600 \text{ mm}$$

$$A_3 = 20 \times 20 = 400 \text{ mm}^2, \quad l_3 = 400 \text{ mm}$$

$$E = 1.5 \times 10^5 \text{ N/mm}^2$$



$$\delta l = \frac{1}{E} * \left[\frac{P_1 l_1}{A_1} + \frac{P_2 l_2}{A_2} + \frac{P_3 l_3}{A_3} \right]$$

$$= \frac{1}{1.5 \times 10^5} * \left[\frac{-20 \times 10^3 \times 500}{400} + \frac{60 \times 10^3 \times 600}{900} - \frac{30 \times 10^3 \times 400}{400} \right]$$

$$= -0.1 \text{ mm} \quad \text{--- decrease in length.}$$

$$\text{So final length} = (500 + 600 + 400) - 0.1$$

$$= 1499.90 \text{ mm}$$



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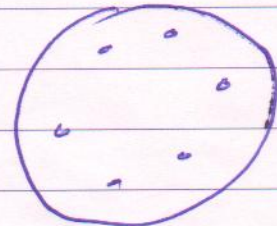
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- ③ A circular RCC column of 250 mm dia. is reinforced with 6 bars of 22 mm dia. The column is carrying a load of 875 kN. Find stresses developed in concrete & steel. $E_s = 2.1 \times 10^5 \text{ N/mm}^2$. $E_c = 0.14 \times 10^5 \text{ N/mm}^2$

→ Dia. of column = 250 mm
6 bars of 22 mm dia.
 $P = 875 \text{ kN}$.



$$A_s = 6 \times \frac{\pi}{4} (22)^2 = 2280.79 \text{ mm}^2$$

$$A_c = \frac{\pi}{4} (250)^2 - 2280.79 = 46806.19 \text{ mm}^2$$

→ equi-strain condition exist, (we know that)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \Rightarrow \sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$\Rightarrow \sigma_s = \frac{2.1 \times 10^5}{0.14 \times 10^5} \times \sigma_c$$

$$\Rightarrow \boxed{\sigma_s = 15 \sigma_c}$$

$$\begin{aligned} \rightarrow P &= P_s + P_c \\ &= \sigma_s A_s + \sigma_c A_c \end{aligned}$$

$$\Rightarrow 875 \times 10^3 = 15 \sigma_c \times 2280.79 + \sigma_c \times 46806.19$$

$$\Rightarrow \boxed{\sigma_c = 10.80 \text{ N/mm}^2} \quad \text{Stress in concrete.}$$



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$$B_s = 15 G_c$$

$$= \underline{162 \text{ N/mm}^2} \text{ stress in steel.}$$



Q. A circular RCC column of 300 mm dia is reinforced with 6 bars of 20 mm dia. The column is carrying a load of 200 kN. Find stresses developed in concrete & steel. $E_s = 2 \times 10^5 \text{ N/mm}^2$, $E_c = 1.5 \times 10^4 \text{ N/mm}^2$



→ Dia of column = 300 mm
 6 bars of 20 mm dia.
 $P = 200 \text{ kN}$

$$A_g = A_c + A_s = 22500 + 600 = 23100 \text{ mm}^2$$

$$A_c = \frac{P}{f_c} \quad A_s = \frac{P}{f_s} \quad \text{Total load } P = 200 \text{ kN}$$

→ strain compatibility (we know that)

$$\frac{e_c}{E_c} = \frac{e_s}{E_s} \Rightarrow \frac{e_c}{1.5 \times 10^4} = \frac{e_s}{2 \times 10^5}$$

$$\Rightarrow e_s = \frac{2 \times 10^5}{1.5 \times 10^4} \times e_c = 13.33 e_c$$

$$\Rightarrow \boxed{e_s = 13.33 e_c}$$

$$\Rightarrow \frac{P}{A_c} + \frac{P}{A_s} = 1.5 e_c \times 22500 + 2 e_s \times 600 = 200 \times 10^3$$

$$\Rightarrow \boxed{f_c = 10.80 \text{ N/mm}^2} \text{ stress in concrete}$$



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- ④ A bar of 16 mm dia. & 12 cm long is subjected to a tensile force of 150 kN. Increase in length & decrease in dia. are observed 0.45 mm & 0.002 mm respectively.

Find: ① Modulus of elasticity.

② Poisson's ratio.

③ Bulk modulus.

④ Modulus of rigidity.

Given data: $d = 16 \text{ mm}$

$$L = 12 \text{ cm} = 120 \text{ mm}$$

$$P = 150 \text{ kN}$$

$$\delta L = 0.45 \text{ mm} = 0.45 \text{ mm}$$

$$\delta d = 0.002 \text{ cm} = 0.002 \text{ mm}$$

$$E = ? \quad \mu = ? \quad K = ? \quad G = ?$$

$$\rightarrow E = \frac{S_L}{d} = \frac{0.45}{120} = 3.75 \times 10^{-3}$$

$$E' = \frac{\delta d}{d} = \frac{0.002}{16} = 1.25 \times 10^{-3}$$

$$\rightarrow \mu = \frac{E'}{E} = \frac{1.25 \times 10^{-3}}{3.75 \times 10^{-3}} = 0.333$$

Poisson's ratio

$$\rightarrow A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$$

$$\rightarrow \sigma = P/A = \frac{150 \times 10^3}{201.06} = 746 \text{ N/mm}^2$$

$$\rightarrow E = \sigma/\epsilon = \frac{746}{3.75 \times 10^{-3}} = \boxed{1.989 \times 10^5 \text{ N/mm}^2}$$

Bulk Modulus:-

$$\mu = \frac{1}{m} = 0.333 \quad \therefore \boxed{m = 3.0}$$

$$K = \frac{mE}{3(m-2)} = \frac{3 \times 1.989 \times 10^5}{3(3-2)} = \boxed{1.989 \times 10^5 \text{ N/mm}^2}$$

* Modulus of Rigidity:-

$$G = \frac{mE}{2(m+1)} = \frac{3 \times 1.989 \times 10^5}{2(3+1)}$$

$$= \boxed{74.58 \times 10^3 \text{ N/mm}^2}$$

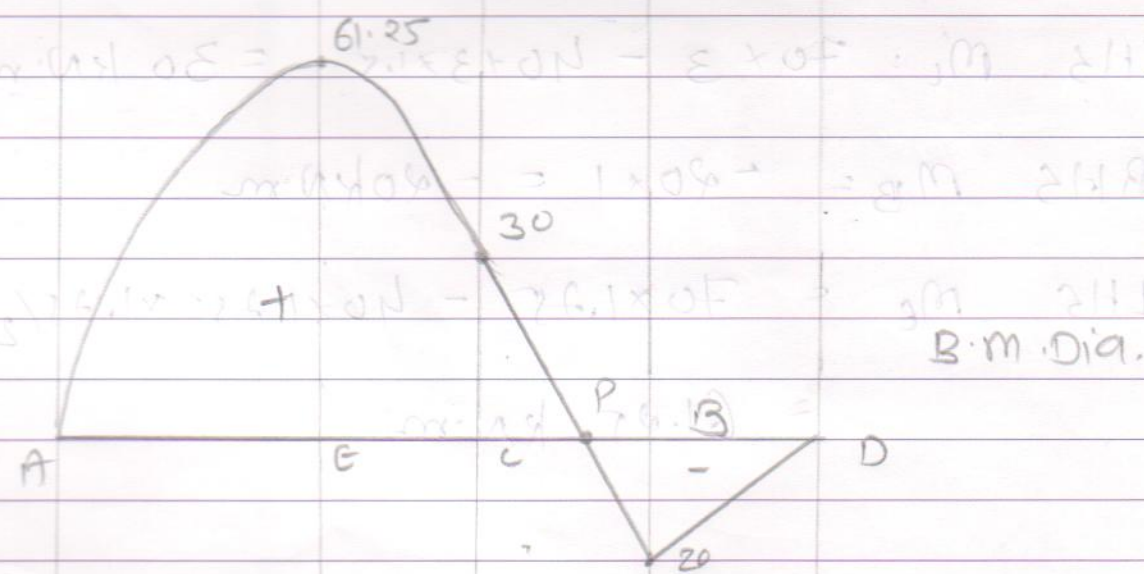
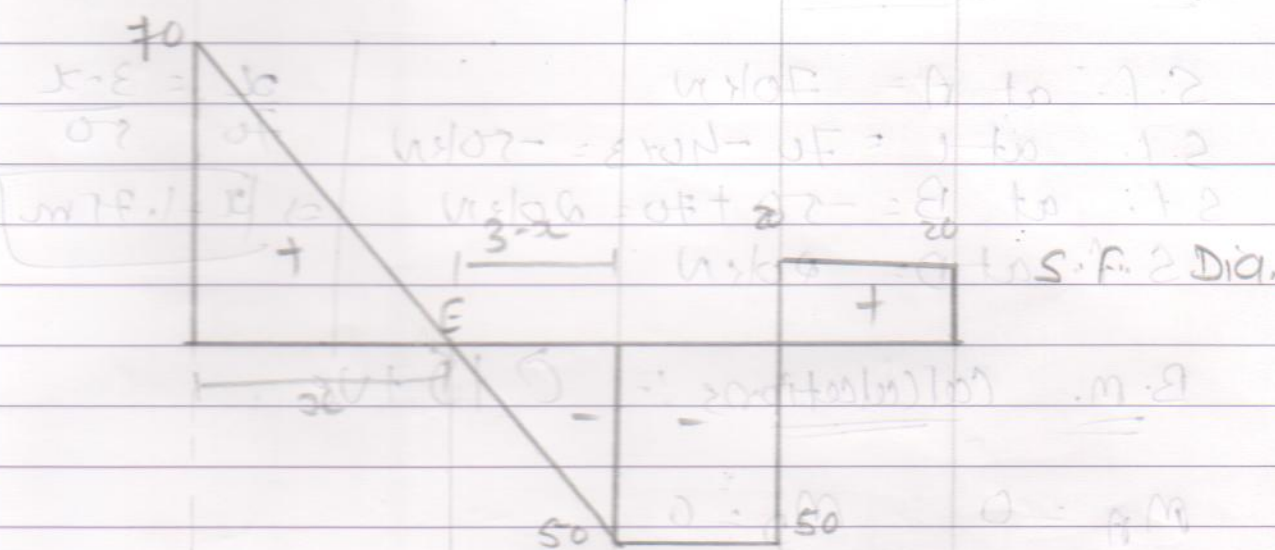
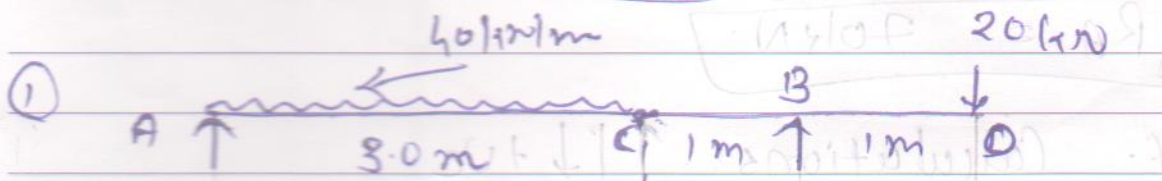
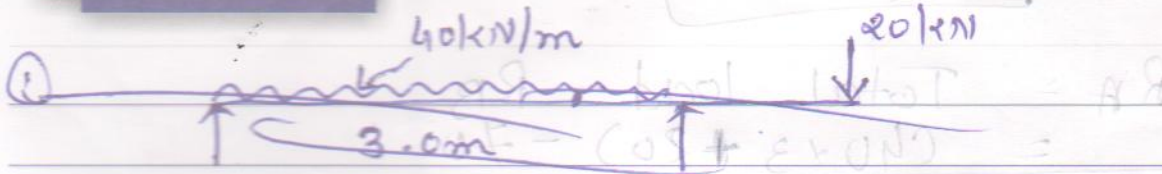


$$\epsilon = \frac{\sigma}{E} = \frac{746}{1.989 \times 10^5} = 3.75 \times 10^{-3}$$

$$\epsilon = \frac{\sigma}{E} = \frac{746}{1.989 \times 10^5} = 3.75 \times 10^{-3}$$

Volume strain

$$\epsilon_v = \frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t} = \epsilon + \epsilon + \epsilon = 3\epsilon$$



→ Taking $M@A = 0$

$$R_B \times 4 = (40 \times 3 \times 1.5) + (20 \times 5)$$

$$\text{So, } \boxed{R_B = 70 \text{ kN}}$$

$$R_A = \text{Total load} - R_B$$
$$= (40 \times 3 + 20) - 70$$

$$\boxed{R_A = 70 \text{ kN}}$$

→ S.f. Calculations: \uparrow | \downarrow +ve

$$\text{S.f. at A} = 70 \text{ kN}$$

$$\text{S.f. at } l = 70 - 40 \times 3 = -50 \text{ kN}$$

$$\text{S.f. at B} = -50 + 70 = 20 \text{ kN}$$

$$\text{S.f. at D} = 0 \text{ kN}$$

$$\frac{x}{70} = \frac{3-x}{50}$$

$$\Rightarrow \boxed{x = 1.75 \text{ m}}$$

→ B.m. Calculations: \curvearrowright | \curvearrowleft +ve

$$M_A = 0 \quad M_D = 0$$

$$\text{LHS } M_C = 70 \times 3 - 40 \times 3 \times 1.5 = 30 \text{ kN}\cdot\text{m}$$

$$\text{RHS } M_B = -20 \times 1 = -20 \text{ kN}\cdot\text{m}$$

$$\text{LHS } M_C = 70 \times 1.75 - 40 \times 1.75 \times 1.75/2$$
$$= 61.25 \text{ kN}\cdot\text{m}$$



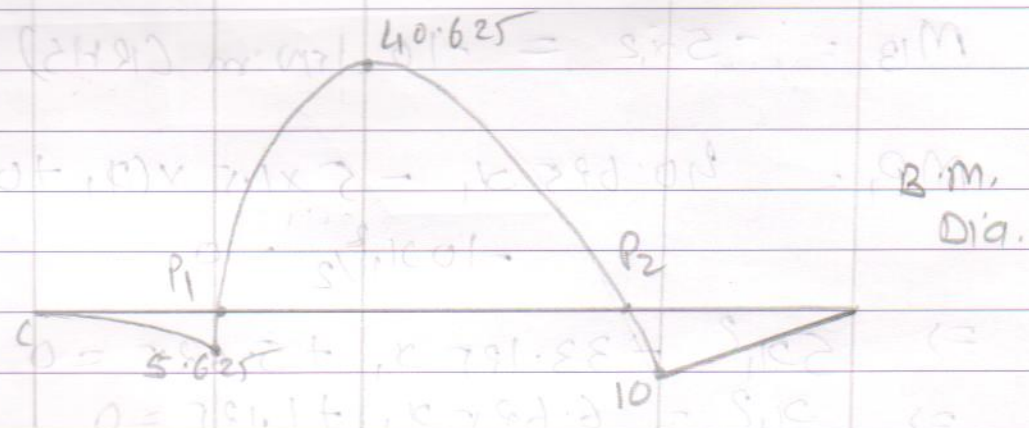
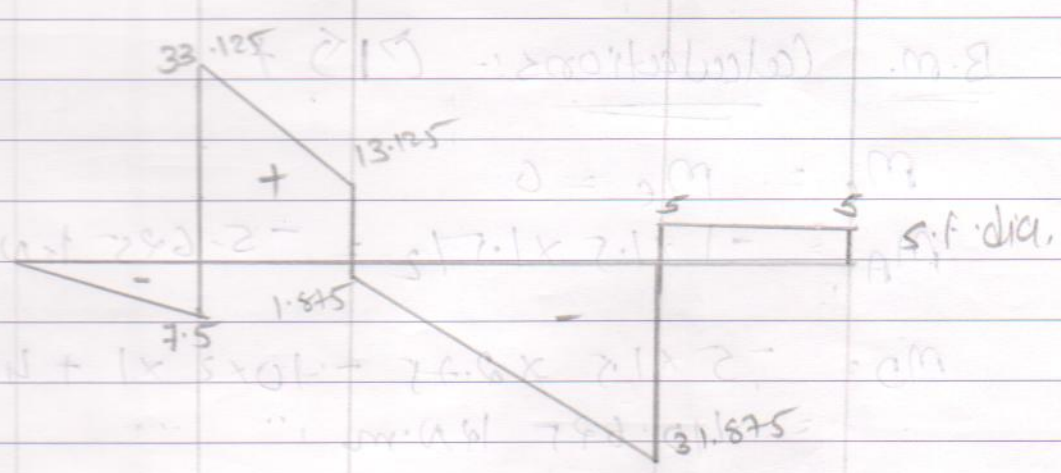
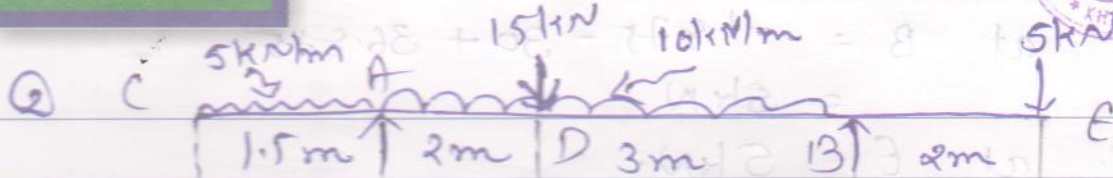
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→ Taking $M @ A = 0$

$$R_B \times 5 + (5 \times 1.5 \times 1.5/2) = (15 \times 2/4) + (10 \times 5) \times 2.5$$

So, $R_B = 36.875 \text{ kN}$

$R_A = 40.625 \text{ kN}$

* S.F. Calculations:- $\uparrow +ve$

S.F. at C = 0

S.F. at A = $(-5 \times 1.5) + (40.625) = 33.125 \text{ kN}$

S.F. at D = $33.125 - 20 - 15 = -1.875 \text{ kN}$

S.F. at B = $-1.875 - 30 + 36.875 = 5 \text{ kN}$

S.F. at E = 5 kN

* B.M. Calculations:- $\curvearrowright +ve$

$M_c = M_e = 0$

$M_A = -1 \times 1.5 \times 1.5 / 2 = -5.625 \text{ kN}\cdot\text{m}$

$M_D = -5 \times 1.5 \times 2.25 - 10 \times 2 \times 1 + 40.625 \times 2 = 40.625 \text{ kN}\cdot\text{m}$

$M_B = -5 \times 2 = -10 \text{ kN}\cdot\text{m (RHIS)}$

$\rightarrow MP_1 = 40.625 x_1 - 5 \times 1.5 \times (x_1 + 0.75) - 10 x_1^2 / 2 = 0$

$\Rightarrow 50 x_1^2 - 33.125 x_1 + 5.625 = 0$

$\Rightarrow x_1^2 - 6.625 x_1 + 1.125 = 0$

So, $x_1 = \frac{6.625 \pm \sqrt{(-6.625)^2 - 4 \times 1 \times 1.125}}{2 \times 1}$

$x_1 = 0.1745 \text{ m}$

$\rightarrow MP_2 = 36.875 x_2 - 5(2 + x_2) - 10 x_2^2 / 2 = 0$

So, $5 x_2^2 - 31.875 x_2 + 10 = 0$

$\Rightarrow x_2^2 - 6.375 x_2 + 2 = 0$

$\therefore x_2 = 0.33 \text{ m}$



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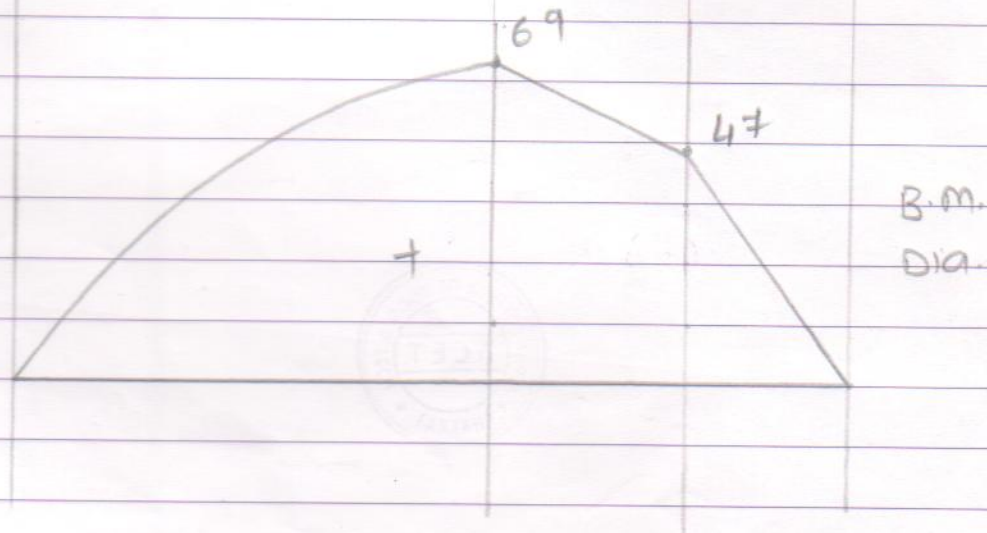
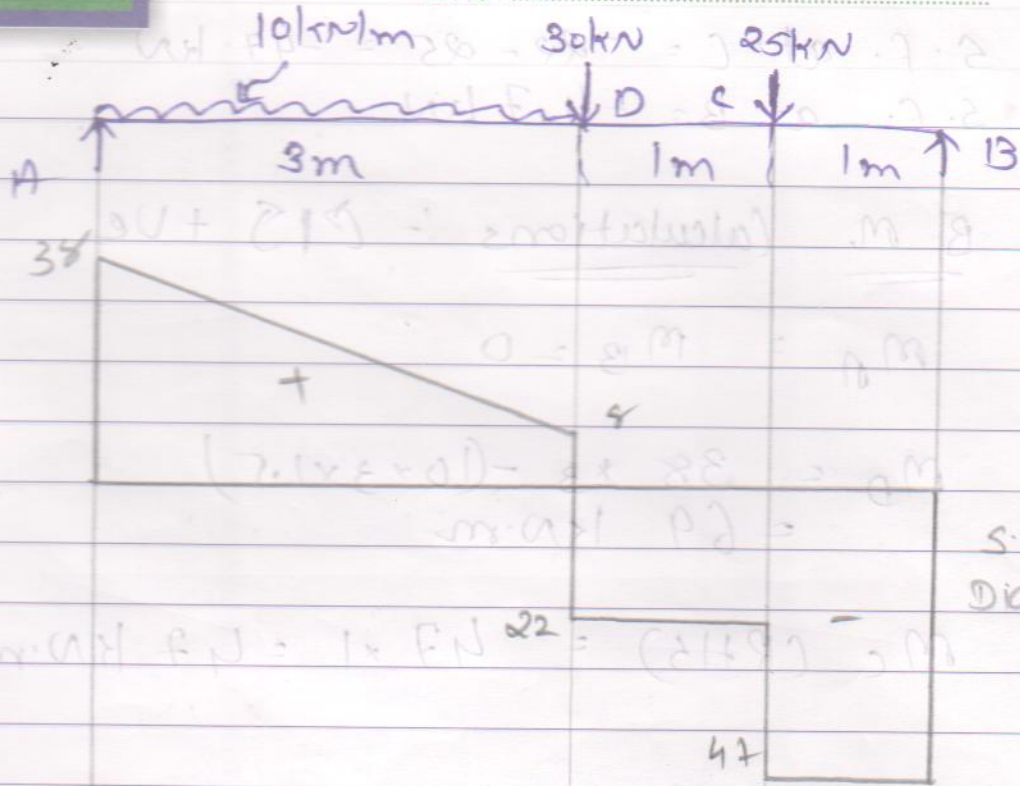
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③



→ Taking $M@A = 0$

$$R_B \times 5 = (10 \times 3 \times 1.5) + (30 \times 3) + (25 \times 4)$$

$$\Rightarrow \boxed{R_B = 47 \text{ kN}}$$

→ S.F. Calculations: $\uparrow / \downarrow +ve$

$$R_A + R_B = \text{Total load}$$

$$\text{S.F. at A} = 38 \text{ kN}$$

$$\text{So } R_A = 38 \text{ kN}$$

$$\text{S.F. at D} = 38 - 30 - 30 \\ = -22 \text{ kN}$$

$$\text{S.F. at C} = -22 - 25 = -47 \text{ kN}$$

$$\text{S.F. at B} = -47 \text{ kN}$$

→ B.M. Calculations :- $\curvearrowright +ve$

$$M_A = M_B = 0$$

$$M_D = 38 \times 3 - (10 \times 3 \times 1.5) \\ = 69 \text{ kN}\cdot\text{m}$$

$$M_C \text{ (RHS)} = 47 \times 1 = 47 \text{ kN}\cdot\text{m}$$



$$R_B + R = (10 \times 3 \times 1.5) + (30 \times 3) + (30 \times 3)$$

$$\Rightarrow R_B = 150 \text{ kN}$$

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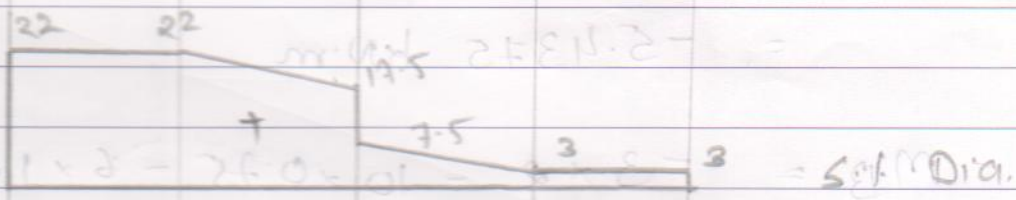
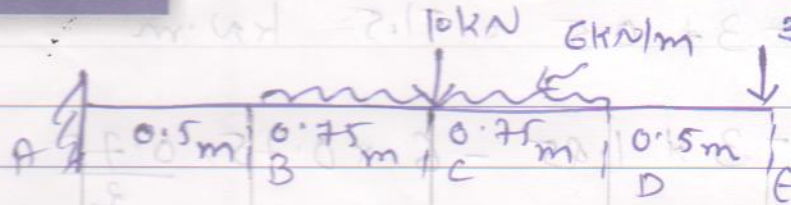
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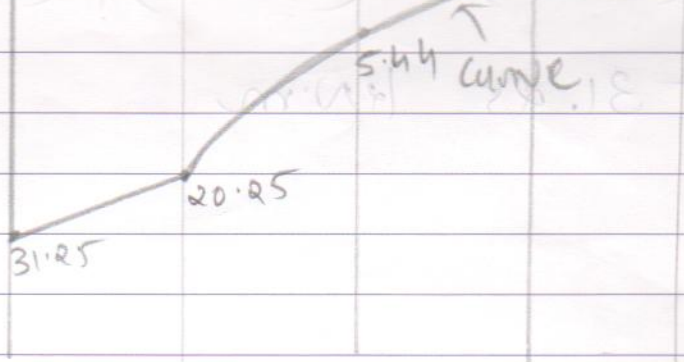
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Bending Moment Diagram (B.M. Dia.) showing the bending moment distribution. The bending moment starts at 31.25 kNm at A, decreases to 20.25 kNm at B, then to 5.44 kNm at C, and finally to 3 kNm at E. The diagram is labeled "B.M. Dia." and includes the calculation $(2.1 \times 2) - (2.1 \times 0.5) = 2.1 \times 1.5$.



→ S.F. Calculations: ↑/↓ +ve

$$\text{S.F. at E} = 3 \text{ kN}$$

$$\text{S.F. at D} = 3 \text{ kN}$$

$$\begin{aligned} \text{S.F. at C} &= 3 + 6 \times 0.75 + 10 \\ &= 17.5 \text{ kN} \end{aligned}$$

$$\text{S.F. at B} = 17.5 + 6 \times 0.75 = 22 \text{ kN}$$

$$\text{S.F. at A} = 22 \text{ kN}$$

* B.m. Calculations :- GID - Ve

$$M_c = 0$$

$$M_D = 3 \times 0.5 = 1.5 \text{ kN}\cdot\text{m}$$

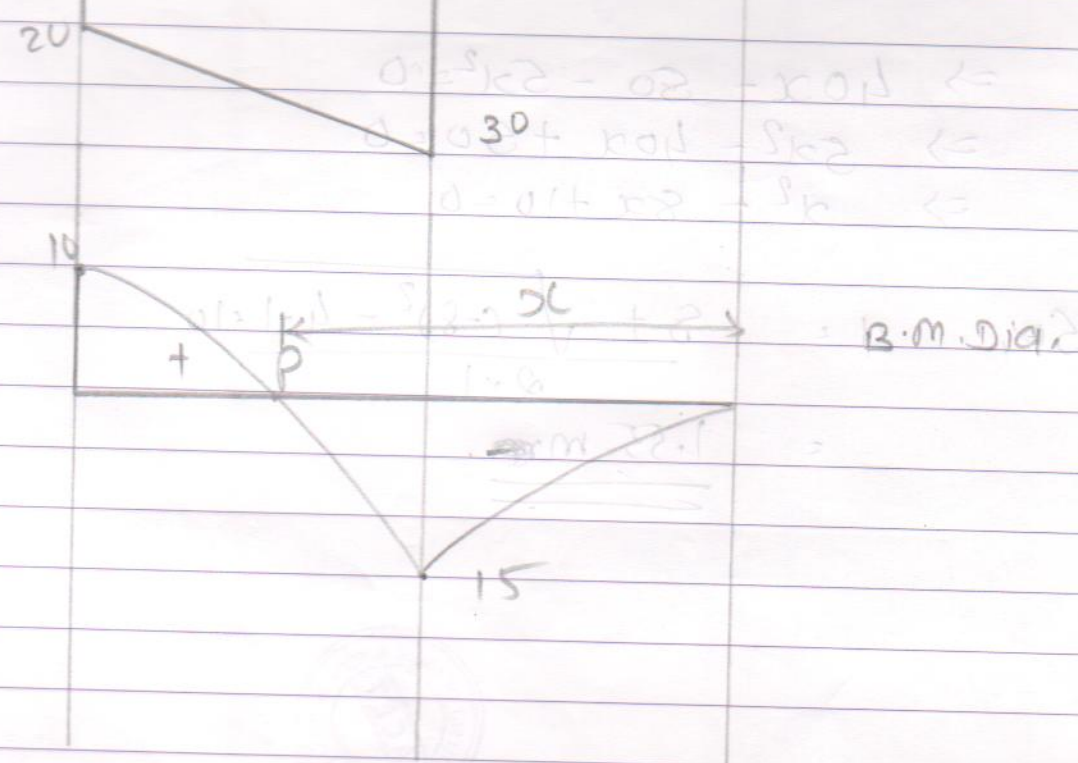
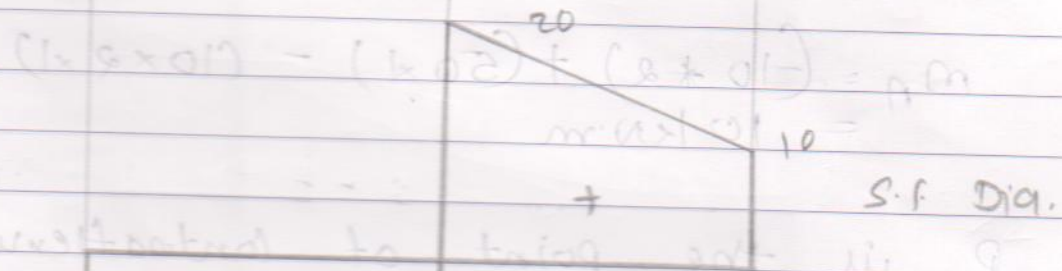
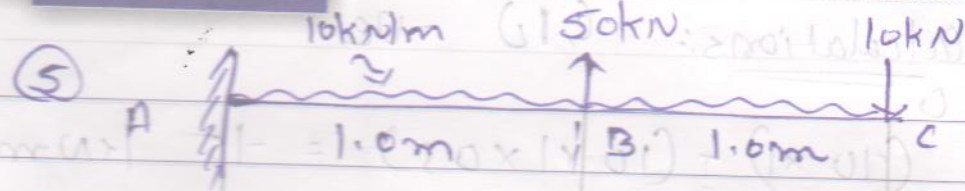
$$M_c = -3 \times 1.25 - 6 \times 0.75 \times \frac{0.75}{2}$$
$$= -5.4375 \text{ kN}\cdot\text{m}$$

$$M_B = -3 \times 2 - 10 \times 0.75 - 6 \times 1.5 \times \frac{2}{2}$$
$$= -20.25 \text{ kN}\cdot\text{m}$$

$$M_A = -(3 \times 2.5) - (10 \times 1.25) - (6 \times 1.5 \times 1.25)$$
$$= -31.25 \text{ kN}\cdot\text{m}$$



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* S.F. Calculations: \uparrow / \downarrow +ve

$$S.F. \text{ at } C = 10 \text{ kN}$$

$$S.F. \text{ at } B = 10 + 10 - 50 = -30 \text{ kN}$$

$$S.F. \text{ at } A = -30 + 10 = -20 \text{ kN}$$

* B.M. Calculations: (10) -ve

$$M_C = 0$$

$$M_B = (10 \times 1) - (10 \times 1 \times 0.5) = -15 \text{ kNm}$$

$$M_A = (-10 \times 2) + (50 \times 1) - (10 \times 2 \times 1) = 10 \text{ kNm}$$

→ P is the point of contraflexure.
एक बिंदु P पर, C की ओर से बनेगा।

$$M_p = -10x + 50(x-1) - 10 \times x \times x/2 = 0$$

$$\Rightarrow 40x - 50 - 5x^2 = 0$$

$$\Rightarrow 5x^2 - 40x + 50 = 0$$

$$\Rightarrow x^2 - 8x + 10 = 0$$

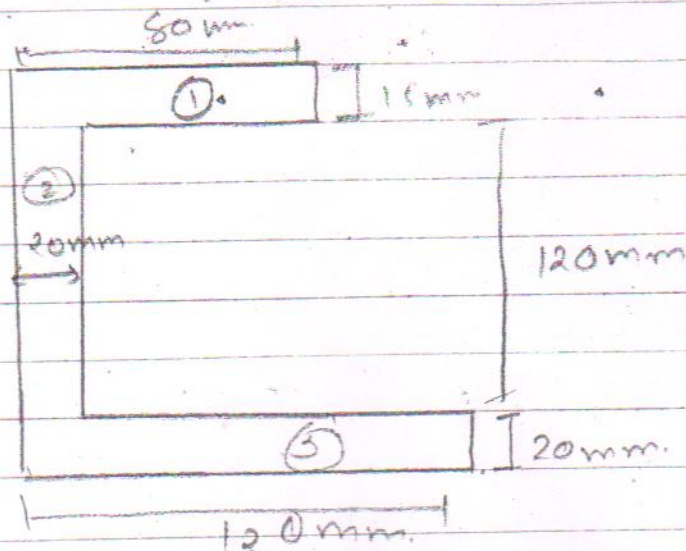
$$\text{So, } x = \frac{8 \pm \sqrt{(8)^2 - 4 \times 1 \times 10}}{2 \times 1}$$

$$= \underline{\underline{1.55 \text{ m}}}$$



Moment of Inertia

Ex.



1st rectangle

$$a_1 = 80 \times 15 = 1200 \text{ mm}^2$$

$$x_1 = 40 \text{ mm}$$

$$y_1 = 20 + 120 + 7.5 = 147.5 \text{ mm}$$

2nd rectangle:-

$$a_2 = 120 \times 20 = 2400 \text{ mm}^2$$

$$x_2 = 10 \text{ mm}$$

$$y_2 = 20 + 60 = 80 \text{ mm}$$

3rd rectangle:-

$$a_3 = 120 \times 20 = 2400 \text{ mm}^2$$

$$x_3 = 60 \text{ mm}$$

$$y_3 = 10 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1200 \times 40) + (2400 \times 10) + (2400 \times 60)}{(1200 + 2400 + 2400)}$$



$$= 36 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= 65.5 \text{ mm}$$

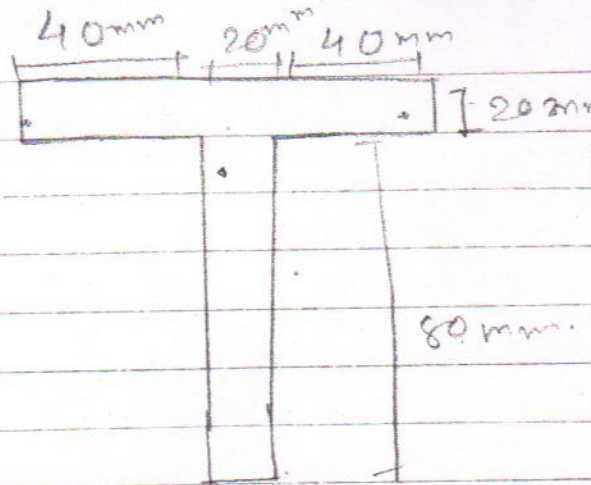
$$\begin{aligned} I_{xx_1} &= I_g + ah^2 \\ &= \frac{80 \times 15^3}{12} + 1200 (147.5 - 65.5)^2 \\ &= 8.09 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_{xx_2} &= I_g + ah^2 \\ &= \frac{20 \times 120^3}{12} + 2400 (80 - 65.5)^2 \\ &= 3.38 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_{xx_3} &= I_g + ah^2 \\ &= \frac{120 \times 20^3}{12} + 2400 (65.5 - 10)^2 \\ &= 7.47 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$I_{xx} = 18.94 \times 10^6 \text{ mm}^4$$

Ex.



T-section is symmetrical about y-axis.

$$\text{So } \bar{x} = \frac{100}{2} = 50 \text{ mm.}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(100 \times 20) \times 90 + (80 \times 20 \times 40)}{(100 \times 20) + (80 \times 20)}$$

$$= 67.78 \text{ mm}$$

$$I_{xx_1} = I_g + ah^2$$
$$= \frac{100 \times 20^3}{12} + 2000 (90 - 67.78)^2$$

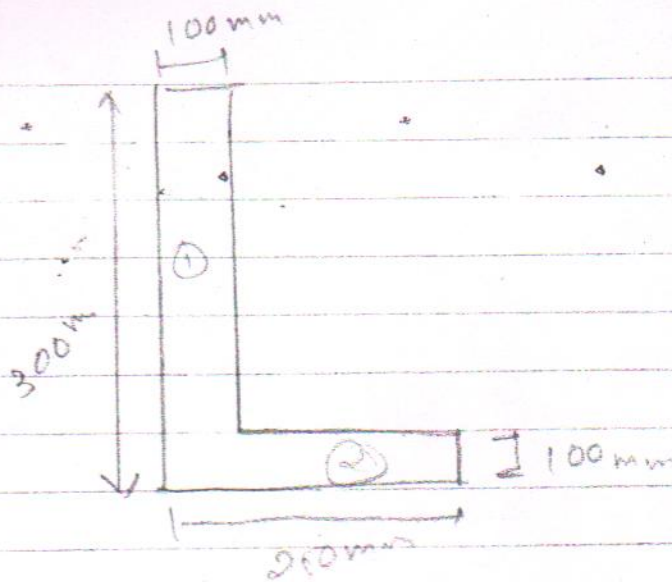
$$= 1.052 \times 10^6 \text{ mm}^4$$

$$I_{xx_2} = I_g + ah^2$$
$$= \frac{20 \times 80^3}{12} + 1600 (67.78 - 40)^2$$

$$= 2.086 \times 10^6 \text{ mm}^4$$

$$I_{xx} = 3.13 \times 10^6 \text{ mm}^4$$

Ex.



$$a_1 = 200 \times 100 = 20,000 \text{ mm}^2$$

$$y_1 = 100 + 100 = 200 \text{ mm}$$

$$a_2 = 250 \times 100 = 25,000 \text{ mm}^2$$

$$y_2 = 50 \text{ mm}$$

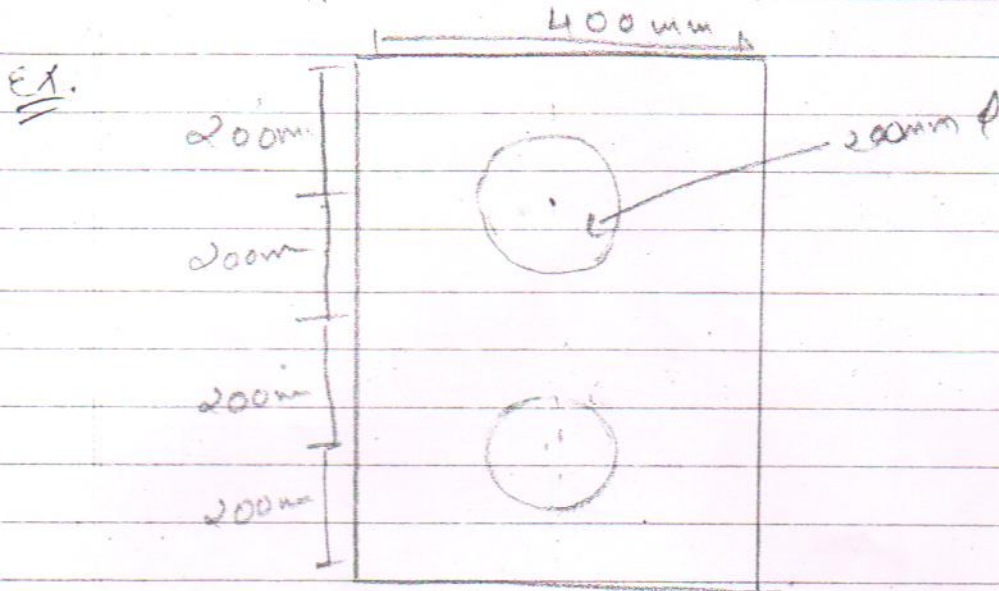
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(20,000 \times 200) + (25,000 \times 50)}{(20,000 + 25,000)}$$

$$= \frac{5,250,000}{45,000}$$

$$\bar{y} = 116.67 \text{ mm}$$

$$\begin{aligned} I_{xx_1} &= I_g + ah^2 \\ &= \frac{100 \times 200^3}{12} + 20,000 (200 - 116.67)^2 \\ &= 205.54 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned}
 I_{xx2} &= I_g + ah^2 \\
 &= \frac{2.50 \times 100^3}{12} + 25,000 (116.67 - 50)^2 \\
 &= 131.95 \times 10^6 \text{ mm}^4 \\
 \therefore I_{xx} &= 337.49 \times 10^6 \text{ mm}^4
 \end{aligned}$$



Rectangle:-

$$\begin{aligned}
 I_{xx1} &= \frac{bd^3}{12} + ah^2 \\
 &= \frac{400 \times 800^3}{12}
 \end{aligned}$$

$$\begin{aligned}
 (\because h=0) \\
 \text{as } \frac{y}{4} = \frac{400}{4} \\
 y_1 = 400
 \end{aligned}$$

$$\begin{aligned}
 \therefore I_{xx1} &= 1.70 \times 10^{10} \text{ mm}^4
 \end{aligned}$$

Circle:-

$$\begin{aligned}
 I_{xx2} &= 2 [I_g + ah^2] \\
 &= 2 \left[\frac{\pi}{64} \times 200^4 + \frac{\pi}{4} \times 200^2 \times 200^2 \right] \\
 &= 2.67 \times 10^9 \text{ mm}^4
 \end{aligned}$$

$$I_{xx} = 1.43 \times 10^{10} \text{ mm}^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{600 \times 400^3}{12} = 4.26 \times 10^9 \text{ mm}^4$$

$$I_{yy_2} = 2 [I_g + ah^2]$$

$$= 2 \left[\frac{\pi}{64} 1200^4 + 2 \times 400^3 \right]$$

$$I_{yy_2} = 157.08 \times 10^6 \text{ mm}^4$$

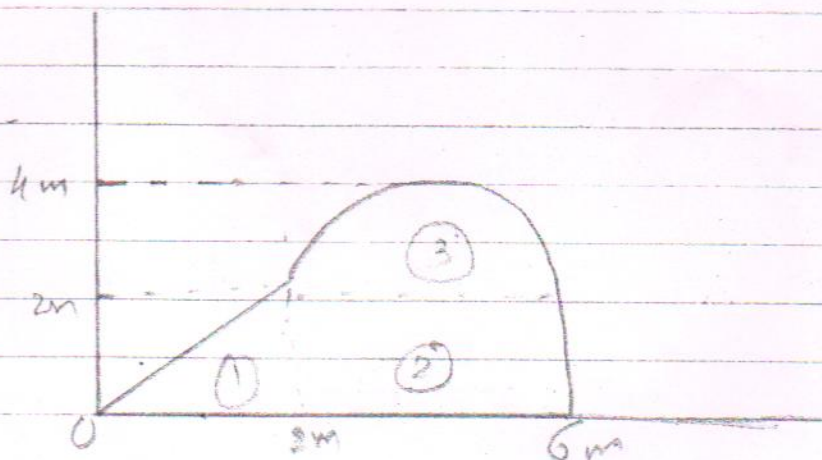
$$I_{yy} = I_{yy_1} - I_{yy_2}$$

$$= 4.108 \times 10^9 \text{ mm}^4$$

$$I_{xx} = I_{xx_1} - I_{xx_2}$$

$$= 1.43 \times 10^{10} \text{ mm}^4$$

Ex.



Triangle :-

$$a_1 = \frac{1}{2} \times 2\text{m} \times 2\text{m} = 2\text{m}$$

$$y_1 = \frac{2}{3} \times 2 = 1.33\text{m}$$

$$y_1 = 1\text{m}$$

Rectangle :-

$$a_2 = 8\text{m} \quad y_2 = 1\text{m}$$

$$x_2 = 4\text{m}$$

semi circle:-

$$a_3 = \frac{1}{2} \pi r^2 = \frac{1}{2} \times 3.14 \times 4 = 6.28 \text{ m}$$

$$x_3 = 4 \text{ m}$$

$$y_3 = 2.84 \text{ m}$$



$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{16.28(2 \times 1) + (8 \times 1) + (6.28 \times 2.84)}{16.28}$$

$$= 1.709 \text{ m}$$

$$I_{xx_1} = I_{cg} + ah^2$$

(triangle) $\frac{bh^3}{12} + 0$

$$= \frac{2 \times 8}{12} = 1.33 \text{ m}^4$$

(i) $I_{xx_2} = \frac{bd^3}{12} + 8(1.709 - 1)^2$

$$= \frac{4 \times 8}{12} + 4.02$$

(ii)

$$= 6.68 \text{ m}^4$$

$$I_{xx_3} = \frac{\pi r^4}{64} + ah^2$$

$$= 12.56 + 8.033$$

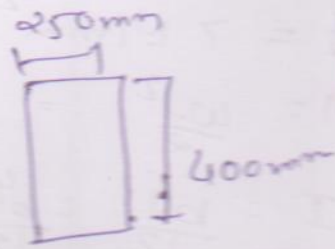
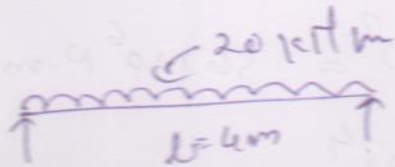
$$= 20.59 \text{ m}^4$$

$$I_{xx} = 28.6 \text{ m}^4$$

Bending Stresses in Beam

Ex.1. A simply supported beam has span 4m and $250\text{mm} \times 400\text{mm}$ cross-section. It carries an UDL of 20 kNm on entire span. Find maximum bending stress and stress on layer 80mm from neutral axis. Draw stress distribution Diagram

Solⁿ



$$M = \frac{wl^2}{8} = \frac{20 \times (4000)^2}{8} = 40 \times 10^6 \text{ N}\cdot\text{mm}$$

$$I = \frac{250 \times 400^3}{12} = 1.33 \times 10^9 \text{ mm}^4$$

$$y = \frac{400}{2} = 200 \text{ mm}$$

$$\frac{M}{I} = \frac{f}{y} \Rightarrow \frac{40 \times 10^6}{1.33 \times 10^9} = \frac{f}{200}$$

$$\boxed{f = 6 \text{ N/mm}^2} \text{ max. bending stress}$$

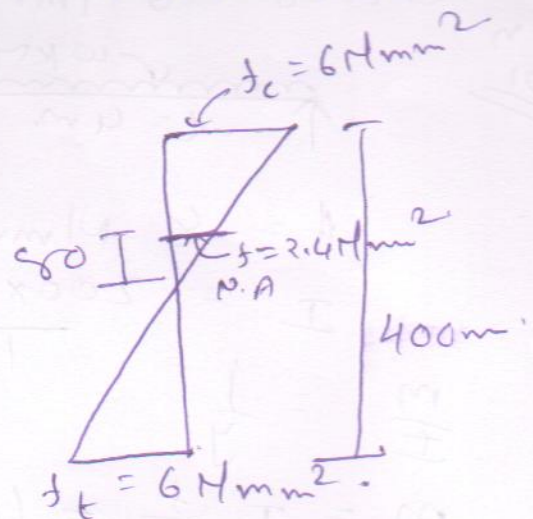
bending stress at 80mm from N.A.

$$y = 80 \text{ mm}$$

$$\frac{M}{I} = \frac{f}{y}$$

$$\frac{40 \times 10^6}{1.33 \times 10^9} = \frac{f}{80}$$

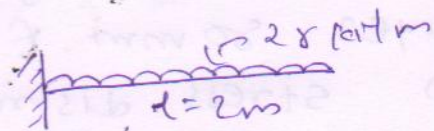
$$\boxed{f = 2.4 \text{ N/mm}^2}$$



Ex.2

A square cantilever beam of span 2m carries a uniformly distributed load of 28 kN/m over entire span. Considering permissible bending stress as 7 N/mm^2 , decide the size of the square beam.

Solⁿ



$$f = 7 \text{ N/mm}^2$$

$$M = \frac{w \cdot l^2}{2} = \frac{28 \times (2000)^2}{2} = 56 \times 10^6 \text{ N}\cdot\text{mm}$$

$$I = \frac{bd^3}{12} = \frac{b^4}{12} = 0.0833b^4$$

$$y = \frac{b}{2} = 0.5b$$

$$\frac{M}{I} = \frac{f}{y} \Rightarrow \frac{56 \times 10^6}{0.0833b^4} = \frac{7}{0.5b}$$

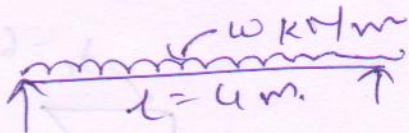
$$\therefore \boxed{b = 363.5 \text{ mm}}$$

\therefore The size of beam is $363.5 \text{ mm} \times 363.5 \text{ mm}$

Ex.3

A wooden beam 200 mm wide & 300 mm deep is simply supported over a span of 4m. If max. bending stress does not exceed 8 N/mm^2 , find max. UDL on beam.

Solⁿ



$$f = 8 \text{ N/mm}^2$$

$$I = \frac{200 \times 300^3}{12} = 45 \times 10^7 \text{ mm}^4$$

$$\frac{M}{I} = \frac{f}{y}$$

$$M = I \cdot \frac{f}{y} = 45 \times 10^7 \times \frac{8}{150} = 24 \times 10^6 \text{ N}\cdot\text{mm}$$

For simply supported beam with u.d.l

$$M = \frac{wl^2}{8}$$

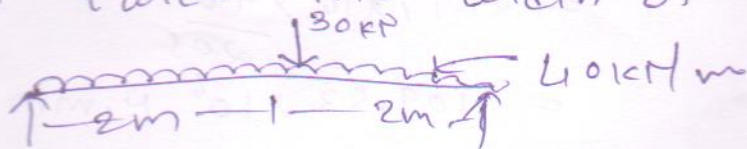
$$24 \times 10^6 = \frac{w \times 4000^2}{8} = 12 \text{ N/mm}$$

$$w = 12 \text{ kN/m}$$



Ex. 4 A rectangular section is used as a simply supported beam of 4m length. It carries u.d.l of 40 kN/m on full length along with a central point load 30 kN. Find width and depth of the section if max. bending stress in the beam is not to exceed 250 N/mm². The depth of section is twice the width of the section.

solⁿ



$$f = 250 \text{ N/mm}^2$$

$$d = 2b \text{ (given)}$$

$$M = \frac{wl}{4} + \frac{wl^2}{8}$$

$$= \frac{30 \times 4}{4} + \frac{40 \times 4^2}{8} = 110 \times 10^6 \text{ Nmm}$$

$$I = \frac{bd^3}{12} = \frac{b(2b)^3}{12} = 0.67b^4$$

$$y = \frac{d}{2} = \frac{2b}{2} = b$$

$$\frac{M}{I} = \frac{f}{y}$$

$$\frac{110 \times 10^6}{0.67b^4} = \frac{250}{b}$$

$$b^3 = 6567.41$$

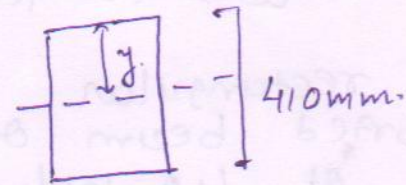
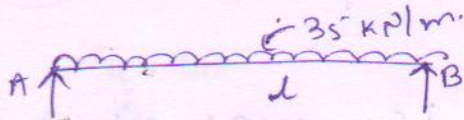
$$b = 86.92 \text{ mm}$$

$$d = 2b = 2 \times 86.92$$

$$d = 173.84 \text{ mm}$$

EX.5. A Beam 410mm deep has M.I, $I_{xx} = 25017 \text{ cm}^4$. Its section is used as a simply supported beam to carry u.d.l of 35 kN/m on its entire length and maximum bending stress is not to exceed 90 N/mm^2 . Find the span of beam.

Solⁿ



$$f = 90 \text{ N/mm}^2$$

$$y = \frac{410}{2} = 205 \text{ mm}$$

$$I_{xx} = 25017 \times 10^4 \text{ mm}^4$$

$$\frac{M}{I} = \frac{f}{y} \Rightarrow M = I \cdot \frac{f}{y}$$

$$= 25017 \times 10^4 \times \frac{90}{205}$$

$$= 109.83 \times 10^6 \text{ N.mm}$$

$$M = \frac{wl^2}{8}$$

$$109.83 \times 10^6 = \frac{35 \times l^2}{8}$$

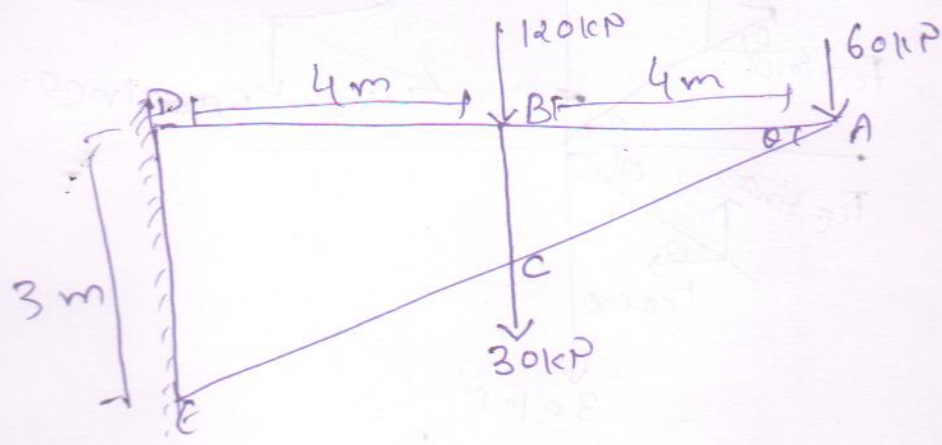
$$\therefore l^2 = 25104000$$

$$\boxed{l = 5010 \text{ mm}}$$



Trouss

Ex.1 Find forces in all the member of a cantilever truss as shown in fig.



Solⁿ

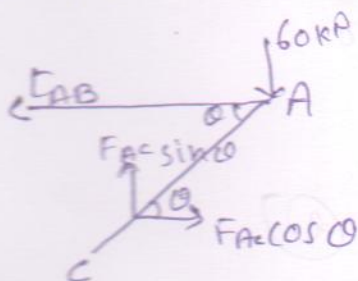
$$\tan \theta = \frac{3}{8} = 0.375 \quad \Rightarrow \quad \theta = 20.55^\circ$$

$$\sin \theta = 0.351$$

$$\cos \theta = 0.936$$

Joint - A:-

Assume tensile force in AB and compressive force in AC.



$$\Sigma V = 0$$

$$\therefore F_{AC} \sin \theta = 60$$

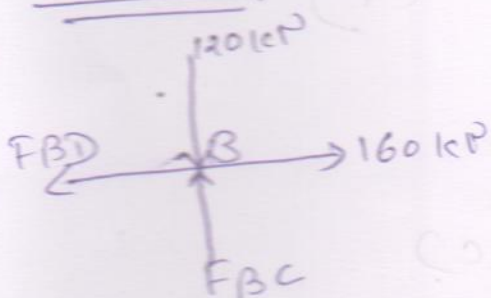
$$F_{AC} = \frac{60}{0.351}$$

$$= 170.94 \text{ kN (C)}$$

$$\Sigma H = 0$$

$$F_{AB} = F_{AC} \cos \theta = 170.94 \times 0.936 = 160 \text{ kN (T)}$$

Joint - B



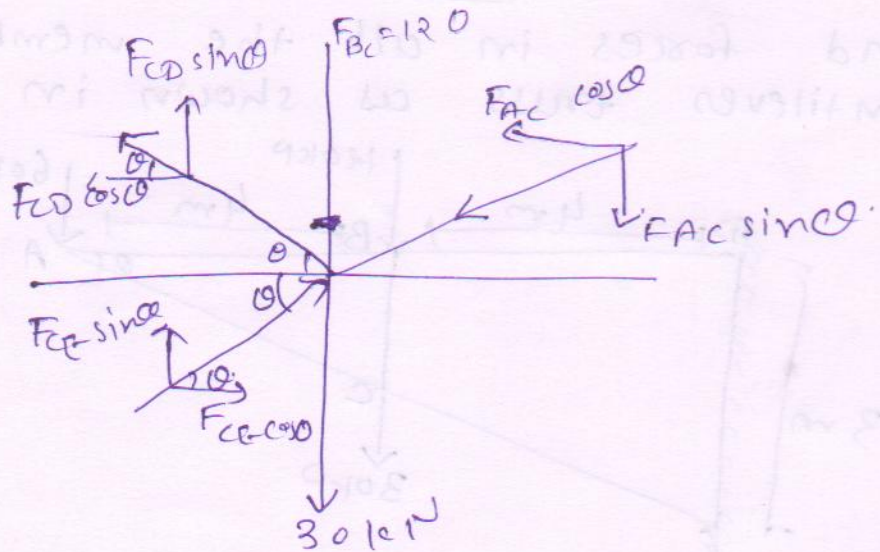
$$\Sigma V = 0$$

$$F_{BC} = 120 \text{ kN (C)}$$

$$\Sigma H = 0$$

$$F_{BD} = 160 \text{ kN (T)}$$

Joint-c



$$\sum V = 0$$

$$\therefore F_{CD} \sin \theta + F_{CE} \sin \theta = 120 + 30 + F_{AC} \sin \theta$$

$$\therefore F_{CD} \times 0.351 + F_{CE} \times 0.351 = 120 + 30 + (170.94 \times 0.351)$$

$$\therefore F_{CD} \times 0.351 + F_{CE} \times 0.351 = 210$$

$$F_{CD} + F_{CE} = 598.29 \quad \text{--- (1)}$$

$$\sum H = 0$$

$$F_{AC} \cos \theta + F_{CD} \cos \theta = F_{CE} \cos \theta$$

$$\therefore F_{AC} + F_{CD} = F_{CE}$$

$$\therefore F_{CD} - F_{CE} = -F_{AC}$$

$$\therefore F_{CD} - F_{CE} = -170.94 \quad \text{--- (2)}$$

solving eqⁿ (1) & (2)

$$F_{CD} + F_{CE} = 598.29$$

$$F_{CD} - F_{CE} = -170.94$$

$$\hline 2 F_{CD} = 427.35$$

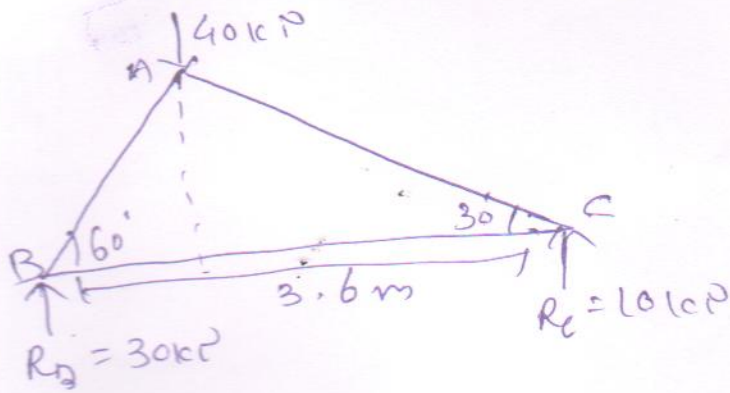
$$\boxed{F_{CD} = 213.67 \text{ kN}} \quad \text{(c1)}$$

$$F_{CD} + F_{CE} = 598.29$$

$$213.67 + F_{CE} = 598.29$$

$$\boxed{F_{CE} = -384.62 \text{ kN}} \quad \text{(c2)}$$

Ex. 2 Find forces in all the members of the frame shown by method of joints.



Solⁿ From ΔABC , $\cos 60^\circ = \frac{AB}{BC}$
 $AB = BC \cos 60^\circ = 3.6 \times 0.5 = 1.8 \text{ m}$

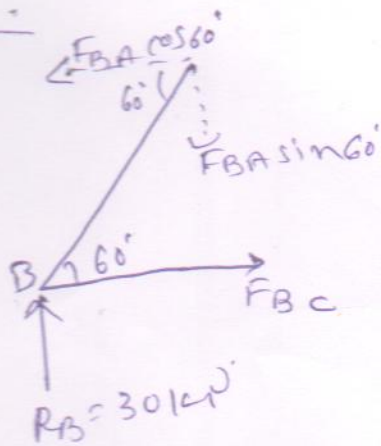
from ΔADB , $\cos 60^\circ = \frac{BD}{AB}$

$\therefore BD = AB \cos 60^\circ = 1.8 \times 0.5 = 0.9 \text{ m}$

Taking moment @ B
 $R_C \times 3.6 = 40 \times 0.9$
 $R_B = 30 \text{ kN}$

$\Rightarrow R_C = 10 \text{ kN} \uparrow$

Joint - B.



$\sum V = 0$

$\therefore F_{BA} \sin 60^\circ = 30$

$\therefore F_{BA} = 36.64 \text{ kN (C)}$

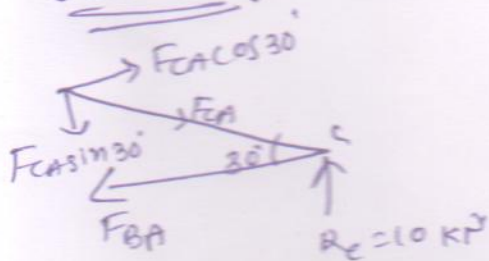
$\sum H = 0$

$F_{BC} = F_{BA} \cos 60^\circ$

$= 36.64 \times 0.5$

$= 17.32 \text{ kN (T)}$

Joint - C.



$\sum V = 0$

$F_{CA} \sin 30^\circ = 10$

$F_{CA} = 20 \text{ kN (C)}$